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# Chiral field theories from conifolds

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Abstract: We discuss the geometric engineering and large $N$ transition for an $\mathcal{N}=1$ $\mathrm{U}(N)$ chiral gauge theory with one adjoint, one conjugate symmetric, one antisymmetric and eight fundamental chiral multiplets. Our IIB realization involves an orientifold of a non-compact Calabi-Yau $A_2$ fibration, together with D5-branes wrapping the exceptional curves of its resolution as well as the orientifold fixed locus. We give a detailed discussion of this background and of its relation to the Hanany-Witten realization of the same theory. In particular, we argue that the T-duality relating the two constructions maps the $\mathbb{Z}_2$ orientifold of the Hanany-Witten realization into a $\mathbb{Z}_4$ orientifold in type IIB. We also discuss the related engineering of theories with SO/Sp gauge groups and symmetric or antisymmetric matter.

Keywords: D-branes, Brane Dynamics in Gauge Theories.



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## Contents





## 1. Introduction

String theory provides nontrivial information about supersymmetric gauge theories by means of geometric engineering. An interesting class of models is based on local conifold geometries [1]–[4]. These arise in IIB string theory on a resolved ADE fibration over the complex plane, with D5-branes partially wrapped on the exceptional divisors. On the common non-compact part of the world volumes, one obtains an $\mathcal{N}=1$ supersymmetric field theory in four dimensions, whose precise nature is dictated by the Calabi-Yau geometry. The gauge group of such models is a product of $\mathrm{U}(N)$ factors, with $N$ fixed by the number of partially wrapped D5-branes. It has been argued that at large $N$ but fixed $g_{\mathrm{string}}N$ such models undergo a transition in which the resolved geometry is replaced by the background obtained through a conifold transition, realized by blowing down all exceptional curves and smoothing out the resulting singular fibration. The exact effective superpotential of the gauge theory can then be calculated by evaluating period integrals of the deformed geometry.

This construction leads to the conjecture that the effective superpotential of a confining $\mathcal{N} = 1$ supersymmetric gauge theory can be computed using a certain holomorphic [8] matrix model [5]–[8]. In a related development it was shown that the loop equations of this model can be recovered by studying certain generalized Konishi anomalies of the gauge theory [9].

The approach described above leads to theories whose gauge group is a product of $\mathrm{U}(N)$ factors, with matter in adjoint or bifundamental chiral multiplets. To engineer more general models, one can consider orientifolds of such backgrounds, possibly supplemented by the introduction of further D-branes. Such extensions of the framework of [1]–[4] have been much less studied. The simplest examples of $\mathbb{Z}_2$ orientifolds where considered in [15]–[17], while a considerably more complicated system was recently analyzed in [18, 19].

It is known that orientifolds in the presence of D-branes can lead to chiral field theories under specific conditions. For IIA backgrounds, the standard example is provided by the work of [11]–[13], which leads to an $\mathcal{N} = 1$ gauge theory with net chirality through a combination of subtle phenomena. More precisely, it was shown in [11]–[13] that one can obtain an $\mathcal{N} = 1$ $\mathrm{U}(N)$ supersymmetric model with one adjoint chiral multiplet, eight fundamental multiplets as well as multiplets transforming in the antisymmetric and conjugate symmetric representations of the gauge group. The IIA realization of this model involves a certain $\mathbb{Z}_2$ orientifold of a Hanany-Witten configuration with three NS5-branes and N pairs of D4-branes, with the addition of half D6-branes along the O6 plane, taken such that they end on the central NS5-brane.[1] The Konishi anomalies of this chiral theory were recently studied in [10], where it was shown that they are reproduced by a certain holomorphic matrix model. In the present paper, we use the methods of [14] to propose a novel IIB realization of this theory as a $\mathbb{Z}_4$ orientifold of a certain non-compact Calabi-Yau $A_2$ fibration with D5-branes wrapping the exceptional fibers of the resolution, and supplemented by the introduction of eight fractional D-branes lying along the orientifold fixed locus (the latter branes are fractional with respect to the $\mathbb{Z}_2$ orbifold group of the orientifold group).

The paper is organized as follows. In section 2 we introduce our field-theoretic model. In section 3 we give a detailed construction of the relevant IIB background. In particular, we present global and local models of the resolved geometry following [14] as well as an explicit construction of the T-duality which maps this background to the Hanany-Witten construction. Our proposal for the T-duality map follows an ansatz first considered in [19] and predicts that the $\mathbb{Z}_2$ orientifold of [11] is mapped to a $\mathbb{Z}_4$ orientifold in IIB. In section 4 we consider the large N transition and effective superpotential. Section 5 discusses the geometric engineering of the related theories with SO / Sp gauge groups and symmet-

[1] A IIB realization of this system was discussed in [20]. This is obtained by performing a certain T-duality which differs from the one we shall consider in this paper.



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ric/antisymmetric matter. Section A contains our conclusions. In the appendices A–C we discuss the relation to general log-normalizable Calabi-Yau $A_2$ fibrations, a fractional brane construction related to our engineering and the orientifold projection on the Chan-Paton factors.

## 2. The field theory model

Our $\mathcal{N}=1$ model contains chiral multiplets $\Phi, A, S$ in the adjoint, antisymmetric and conjugate symmetric representations of $\mathrm{U}(N)$ as well as eight quarks $Q_1, \ldots, Q_8$ in the fundamental representation. We consider the tree-level superpotential:

$$W_{\text{tree}} = \mathrm{tr}\,[W(\Phi) + S\Phi A] + \sum_{f=1}^{8} Q_f^T S Q_f\,, \tag{2.1}$$

where:

$$W(z) = \sum_{j=1}^{d+1} \frac{t_j}{j} z^j \tag{2.2}$$

is a complex polynomial of degree $d+1$. The fields are constrained by $S^T = S, A^T = -A$ and the gauge transformations take the form:

$$\Phi \to U\Phi U^\dagger\,, \qquad S \to \bar{U} S U^\dagger\,, \qquad A \to U A U^T\,, \qquad Q_f \to U Q_f\,, \tag{2.3}$$

where $U$ is valued in $\mathrm{U}(N)$. The restriction to eight quark flavors follows from cancellation of the chiral anomaly. This model was studied in [10] through the method of generalized Konishi anomalies, which allows for a proof of the Dijkgraaf-Vafa correspondence in this situation.

As discussed in [11]–[13], our model can be obtained in IIA string theory through a Hanany-Witten configuration involving a $\mathbb{Z}_2$ orientifold of a system of NS5, D4 and half D6-branes (this construction is recalled — and slightly extended — in subsection 3.2). Our first purpose is to determine the geometric engineering of this field theory and extract its relation with the Hanany-Witten construction. We shall find that the engineering of our system is quite nontrivial, and in particular it involves $\mathbb{Z}_4$ orientifolds of resolved Calabi-Yau $A_2$ fibrations. Such backgrounds contain D5-branes wrapped over the exceptional fibers of the resolution, as well as D5-branes stretching along the fixed point set of the orientifold 5-plane.[2]

## 3. Chiral field theories from geometric engineering

In this section we engineer our model by considering a $\mathbb{Z}_4$ orientifold of a IIB background with D5-branes. We also relate this to the Hanany-Witten description of [11]–[13] by applying T-duality with respect to a certain U(1) action and using an ansatz for the dual coordinates originally considered in [19].

[2] In contrast the T-duality considered in [20, 21] results in a $\mathbb{Z}_2$ orientifold 7-plane together with a collection of D3-branes and D7-branes.



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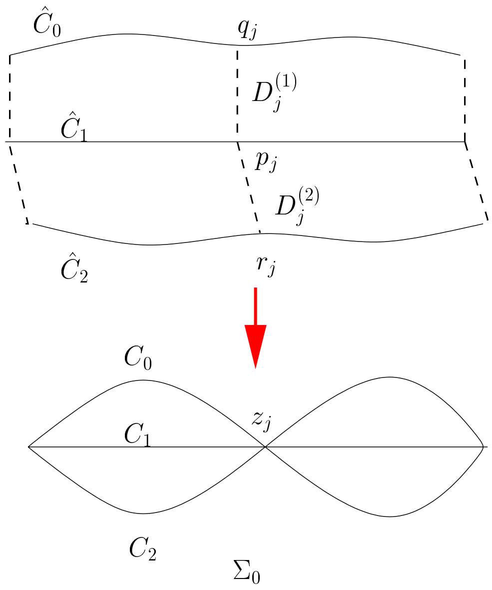


**Figure 1:** The degenerate curve $\Sigma_0$ and its transform $\hat{\Sigma}$ (to be discussed in the next subsection). The orientifold action fixes $\hat{C}_1$ and flips $\hat{C}_0$ and $\hat{C}_2$.

## 3.1 The IIB background

Our starting point is a Calabi-Yau $A_2$ fibration, chosen to admit a certain holomorphic $\mathbb{Z}_4$ action. We shall first consider the singular limit of this fibration, then provide global and local models of its resolution following [14].

### 3.1.1 The singular limit $X_0$

Consider the Calabi-Yau hypersurface $X_0$ defined by the equation:

$$xy = u\left(u - W'(z)\right)\left(u + W'(z)\right), \tag{3.1}$$

where $z, u, x, y$ are the affine coordinates of $\mathbb{C}^4$ and $W$ is a polynomial of degree $d+1$ with complex coefficients. Throughout this section, we restrict to the generic case when the derivative $W'$ has only simple roots, which we denote by $z_j$. The index $j$ runs from 1 to $d$.

The space (3.1) is a singular $A_2$ fibration over the $z$-plane, which admits a 3-section $\Sigma_0$ obtained by requiring $x = y = 0$. Together with (3.1) this gives the planar curve with equation:

$$u\left(u - W'(z)\right)\left(u + W'(z)\right) = 0\,. \tag{3.2}$$

Thus $\Sigma_0$ has three rational components, which we index as follows:

$$C_0 : \; u = -W'(z)\,, \qquad C_1 : \; u = 0\,, \qquad C_2 : \; u = +W'(z)\,. \tag{3.3}$$

One easily checks that the components meet only when $u = 0$ and $z$ coincides with a critical point of $W$. In fact $\Sigma_0$ has triple points sitting at $u = 0$, $z = z_j$ and no other singular points (see figure 1).

The total space (3.1) also has $d$ singular points, which coincide with the triple points of $\Sigma_0$. The fiber sitting above $z_j$ is a standard $A_2$ singularity:

$$xy = u^3 \,. \tag{3.4}$$

When $z$ is away from all $z_j$, the fiber $X_0(z)$ is a smooth deformation of (3.4). Resolving each singularity of $X_0$ leads to a smooth space $\hat{X}$ which we describe in the next subsection.

Below, we shall be interested in the U(1) action on $X_0$ given by:

$$\rho_0(\theta): \quad (z,u,x,y) \to (z,u,e^{i\theta}x\,,e^{-i\theta}y)\,, \tag{3.5}$$

whose fixed point set coincides with the multisection $\Sigma_0$. We also consider the bi-holomorphism:

$$\kappa_0: \quad (z,u,x,y) \longrightarrow (z,-u,-y,x)\,, \tag{3.6}$$

whose fixed point set $O_0$ coincides with the central component $C_1$. We shall use the later to define a $\mathbb{Z}_4$ orientifold of our background, as explained in more detail below. The square of the generator (3.6) is given by:

$$\kappa_0^2: \quad (z,u,x,y) \longrightarrow (z,u,-x,-y) \tag{3.7}$$

and thus $\kappa_0^4 = id$.

**Observation:** the singular space (3.1) can be obtained by starting with a generic Calabi-Yau $A_2$ fibration and requiring invariance under (3.6). This is explained in appendix A.

### 3.1.2 The resolution

Consider the space $\hat{X}$ obtained by resolving each of the singular $A_2$ fibers $X_0(z_j)$. Following [14], we describe it as the complete intersection in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{C}^4$ cut by the equations:

$$\begin{aligned}
\beta_1(u+W'(z)) &= \alpha_1 x \\
\alpha_2(u-W'(z)) &= \beta_2 y \\
\alpha_1\beta_2 u &= \beta_1\alpha_2\,. \\
u(u-W'(z))(u+W'(z)) &= xy
\end{aligned} \tag{3.8}$$

We let $[\alpha_i, \beta_i]$ be the homogeneous coordinates on the two $\mathbb{P}^1$ factors ($i=1,2$). Forgetting these gives the resolution map $\tau: \hat{X} \to X_0$. The exceptional fibers of $\tau$ are given by $z=z_j, u=x=y=0$ and $\alpha_2=0$ or $\beta_1=0$. The case $\alpha_2=0$ defines a rational curve $D_j^{(1)}$ which can be identified with $\mathbb{P}^1[\alpha_1,\beta_1]$, while $\beta_1=0$ gives the curve $D_j^{(2)}$, identified with $\mathbb{P}^1[\alpha_2,\beta_2]$. The rational curves $D_j^{(1)}$ and $D_j^{(2)}$ intersect transversely at a point $p_j$ which sits at $z=z_j, u=x=y=\alpha_2=\beta_1=0$ (see figure 1) . If one views $\hat{X}$ as a fibration over the $z$-plane, then its fiber $\hat{X}(z)$ can be identified with $X_0(z)$ when $W'(z) \neq 0$, and coincides with the minimal resolution of $X_0(z_j)$ when $z=z_j$.

Our next task is to lift the circle action (3.5) and $\mathbb{Z}_4$ generator (3.6) to the resolved space. It is easy to see that (3.5) is covered by the following U(1) action on $\hat{X}$:

$$\hat{\rho}(\theta): \quad (z,u,x,y,[\alpha_1,\beta_1],[\alpha_2,\beta_2]) \longrightarrow (z,u,e^{i\theta}x,e^{-i\theta}y,[e^{-i\theta}\alpha_1,\beta_1],[\alpha_2,e^{i\theta}\beta_2]) \tag{3.9}$$

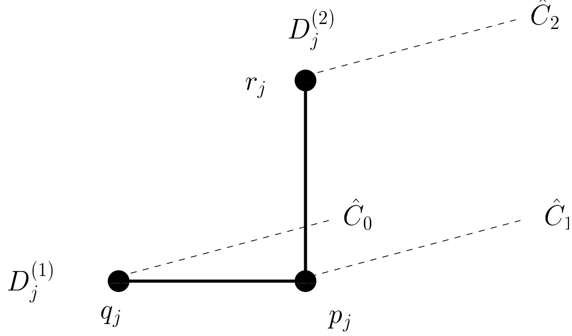


**Figure 2:** Intersection points of $\hat{C}_j$ with the resolved fiber $\hat{X}(z_j)$. We introduce D5-branes partially wrapped on the exceptional curves $D_j^{(1)}$ and $D_j^{(2)}$ and an orientifold 5-plane whose internal part stretches along $\hat{C}_1$. For consistency, we also have eight (fractional) D5-branes stretched along the orientifold locus. These D-branes are fractional because the square of the orientifold generator is nontrivial and induces a $\mathbb{Z}_2$ orbifold action.

while (3.6) lifts to:

$$\hat{\kappa}: \quad (z, u, x, y, [\alpha_1, \beta_1], [\alpha_2, \beta_2]) \longrightarrow (z, -u, -y, x, [\beta_2, \alpha_2], [-\beta_1, \alpha_1])\,. \tag{3.10}$$

The fixed point set of (3.9) has three rational components $\hat{C}_0$, $\hat{C}_1$ and $\hat{C}_2$ which project to the curves $C_0, C_1$ and $C_2$ via the resolution map. These are given by $x = y = 0$ and:

$$\begin{aligned}
\hat{C}_0 &: \; u + W'(z) = \alpha_1 = \alpha_2 = 0 \\
\hat{C}_1 &: \; u = \alpha_2 = \beta_1 = 0 \\
\hat{C}_2 &: \; u - W'(z) = \beta_1 = \beta_2 = 0\,.
\end{aligned}$$

The combination of these curves gives a 3-section $\hat{\Sigma}$ of the fibration $\hat{X}$. When $z$ coincides with a critical point $z_j$, the curves $\hat{C}_1, \hat{C}_2, \hat{C}_3$ meet the resolved fiber $\hat{X}(z_j)$ at three distinct points $q_j, p_j, r_j$ given by $x = y = 0$ and $\alpha_1 = \alpha_2 = 0$, $\alpha_2 = \beta_1 = 0$ and $\beta_1 = \beta_2 = 0$ respectively. The second of these is the intersection of $D_j^{(1)}$ with $D_j^{(2)}$, while the first and last point lie on these exceptional curves respectively (figure 2). As expected, the curves $\hat{C}_j$ are separated inside $\hat{X}(z_j)$, unlike their projections $C_j$ which meet at $x = y = u = 0$, $z = z_j$.

The $\mathbb{Z}_4$ generator (3.10) flips $\hat{C}_0$ and $\hat{C}_2$ while fixing $\hat{C}_1$ — in fact, its fixed point set $\hat{O}$ coincides with the central component $\hat{C}_1$. This action exchanges $D_j^{(1)}$ and $D_j^{(2)}$. As mentioned above, we use (3.10) to define a $\mathbb{Z}_4$ orientifold of our IIB background. In particular, we have an orientifold 5-plane with worldvolume given by $\mathbb{R}^{1,3} \times \hat{C}_1$, which we shall take to be an $O5^-$ plane for agreement with the Hanany-Witten construction discussed below.

One can check that this D-brane configuration reproduces the field content of our chiral theory. One way to establish this is to consider the limit $W' \equiv 0$, with the exceptional $\mathbb{P}^1$'s blown down. Then the background becomes a $\mathbb{Z}_4$ orientifold of a trivial $\mathbb{C}^2/\mathbb{Z}_3$ fibration over the complex plane. The eight D5 branes along the orientifold fixed locus live on the zero section of this fibration, while the D5-branes wrapped over the exceptional $\mathbb{P}^1$'s are represented as D3-branes transverse to the total space of the fibration and delocalized along $z$. This allows one to implement the orientifold action by considering a projection originally used in [20] (albeit in a different context). The details of this analysis can be found in appendix B. An alternate derivation of the massless spectrum (which does not involve taking the fractional brane limit) is given in appendix C. In this second approach, the symmetric and antisymmetric multiplets arise from strings stretching between the D5-branes wrapping the exceptional fibers, while the fundamentals arise from strings stretching between these branes and the eight fractional D5 branes wrapped over $\hat{C}_1$ (the massless states of all these types are localized at the intersection points $p_j$). Finally, the adjoint field arise from strings stretching between the D5 branes wrapping the exceptional fibers $D_j^{(\alpha)}$. Their massless components correspond to strings stretching between branes wrapping the same exceptional curve, which give rise to massless states living on that curve.

### 3.1.3 A local model of the resolution

As in [14], we can consider a local model $\tilde{X} \subset \hat{X}$ of the resolution, which is obtained by gluing three copies $\mathcal{U}_j$ $(j = 0, \ldots, 2)$ of $\mathbb{C}^3$ (with affine coordinates $x_j, u_j, z_j$) according to the identifications:

$$\begin{aligned}
(z_1, x_1, u_1) &= \left(z_0, \frac{1}{u_0}, x_0 u_0^2 - W'(z_0) u_0\right) \\
(z_2, x_2, u_2) &= \left(z_1, \frac{1}{u_1}, x_1 u_1^2 - W'(z_1) u_1\right).
\end{aligned} \tag{3.11}$$

The restricted projection $\tau : \tilde{X} \to X$ is given by:

$$\begin{aligned}
(z,u,x,y) &= \big(z_0, x_0 u_0 - W'(z_0), x_0, u_0 [x_0 u_0 - W'(z_0)][x_0 u_0 - 2W'(z_0)]\big) \\
(z,u,x,y) &= \big(z_1, x_1 u_1, x_1 [x_1 u_1 + W'(z_1)], u_1 [x_1 u_1 - W'(z_1)]\big) \\
(z,u,x,y) &= \big(z_2, x_2 u_2 + W'(z_2), x_2 [x_2 u_2 + W'(z_2)][x_2 u_2 + 2W'(z_2)], u_2\big).
\end{aligned} \tag{3.12}$$

In this description, the exceptional curve $D_j^{(1)}$ sits at $z_1 = z_j$, $u_1 = 0$, while $D_j^{(2)}$ is given by $z_1 = z_j$, $x_1 = 0$.

The orientifold action (3.10) takes the form:

$$\hat{\kappa} : (z_1, x_1, u_1) \to (z_1, u_1, -x_1) \tag{3.13}$$

with fixed point set $\hat{O}$ given by $\hat{C}_1$.

As in [14], the brane construction of [11] arises by performing T-duality with respect to the U(1) action (3.9) on $\hat{X}$, which has the following form in local coordinates:

$$\hat{\rho}(\theta) : \quad (z_j, x_j, u_j) \longrightarrow (z_j, e^{i\theta} x_j, e^{-i\theta} u_j). \tag{3.14}$$

The fixed point locus of (3.14) consists of the three components $\hat{C}_j$, with local equations:

$$\hat{C}_j : \quad u_j = x_j = 0, \tag{3.15}$$

which take the following form in the coordinates of the patch $\mathcal{U}_1$:

$$\begin{aligned} \hat{C}_0 &: \quad x_1 u_1 = -W'(z)\,, \qquad x_1 = \infty \\ \hat{C}_1 &: \quad x_1 = u_1 = 0 \\ \hat{C}_2 &: \quad x_1 u_1 = +W'(z)\,, \qquad u_1 = \infty. \end{aligned} \tag{3.16}$$

The U(1) action stabilizes the exceptional curves $D_j^{(\alpha)}$.

### 3.2 Relation to the orientifolded Hanany-Witten construction

Under T-duality along the orbits of (3.14), the loci $\hat{C}_j$ become three NS5-branes denoted by $\mathcal{N}_j$, while the D5-branes wrapping $D_j^{(\alpha)}$ are mapped into two stacks of $D_4$-branes stretching between them, which we denote by $\mathcal{D}_j^{(\alpha)}$.

To see the T-dual picture explicitly, let us follow [19] by combining $x_1$ and $u_1$ into the quaternion coordinate $X = x_1 + \mathbf{j}u_1$ (where $\mathbf{j}$ is the second imaginary quaternion unit) and notice that the U(1) action (3.14) becomes:

$$X \longrightarrow e^{i\theta} X\,. \tag{3.17}$$

The hyperkahler moment map $\vec{\mu} : \mathbb{H}[X] = \mathbb{C}^2[x_1, u_1] \to \mathbb{R}^3$ of this action presents $\mathbb{C}^2[x_1, u_1]$ as an $S^1$ fibration over $\mathbb{R}^3$ (the fiber collapses to a point at the origin of $\mathbb{R}^3$). Considering the real and complex components of $\vec{\mu}$ allows us to introduce real coordinates $(x^4, \ldots, x^9)$ on the patch $\mathcal{U}_1$ by the equations:

$$\begin{aligned} x^4 + ix^5 &= x_1 u_1 \\ x^6 &= \frac{1}{2}(|x_1|^2 - |u_1|^2) \\ x^7 &= \text{coordinate on the } S^1 \text{ fiber of } \vec{\mu} \\ x^8 + ix^9 &= z_1\,. \end{aligned} \tag{3.18}$$

Then the dual NS5-branes are described by:

$$\begin{aligned} \mathcal{N}_0 &: \quad x^4 + ix^5 = -W'(z)\,, \qquad x^6 = +\infty\,, \qquad x^7 = 0 \\ \mathcal{N}_1 &: \quad x^4 = x^5 = x^6 = x^7 = 0 \\ \mathcal{N}_2 &: \quad x^4 + ix^5 = +W'(z)\,, \qquad x^6 = -\infty\,, \qquad x^7 = 0\,, \end{aligned}$$

while the dual D4-branes $\mathcal{D}_j^{(1)}$ and $\mathcal{D}_j^{(2)}$ are localized at $x^8 + ix^9 = z_j$, $x^4 = x^5 = x^7 = 0$ and extend in the direction $x^6$ between $\mathcal{N}_1$ and $\mathcal{N}_0$, $\mathcal{N}_2$ respectively (i.e. we have $x^6 \geq 0$ for $\mathcal{D}_j^{(1)}$ and $x^6 \leq 0$ for $\mathcal{D}_j^{(2)}$).

Equations (3.18) show that the orientifold action (3.13) fixes $x^8$ and $x^9$ while changing the sign of $x^4$, $x^5$ and $x^6$. The obvious relation:

$$\hat{\kappa} \circ \hat{\rho}(\theta) = \hat{\rho}(-\theta) \circ \hat{\kappa} \tag{3.19}$$

shows that the IIB orientifold action reflects the coordinate $x^7$, which means that the IIA orientifold action must fix this coordinate. Hence T-duality produces an orientifold 6-plane sitting at $x^4 = x^5 = x^6 = 0$, with action:

$$(x^4, x^5, x^6) \to (-x^4, -x^5, -x^6) \tag{3.20}$$



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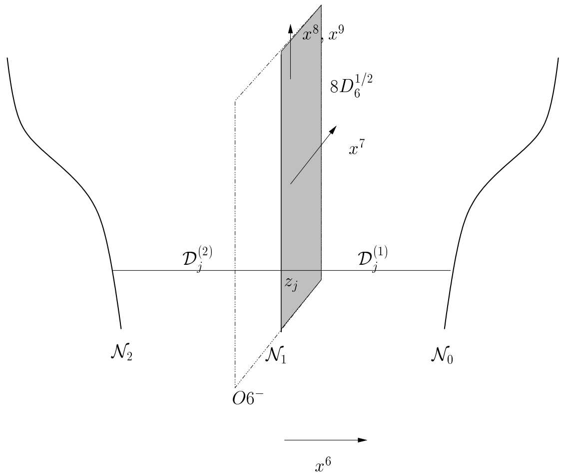


**Figure 3:** The T-dual brane configuration.

on the transverse coordinates. Note that in the IIA construction we obtain a $\mathbb{Z}_2$ orientifold, even though we started with a $\mathbb{Z}_4$ orientifold in IIB. This is because the first two of relations (3.18) are nonlinear. Finally, our eight fractional D5-branes become eight half D6-branes stretching along the O6 plane and ending on the central NS5-brane (figure 3). This recovers the Hanany-Witten configuration of [11].

It is well-known [11]–[13] that this Hanany-Witten configuration produces the matter content of our chiral field theory. More precisely, the effective theory lives on the worldvolume of the D4-branes, which are identified in pairs by the orientifold action, namely $\mathcal{D}_j^{(1)} \equiv \mathcal{D}_j^{(2)}$. This theory has a U($N$) gauge group because the orientifold action identifies gauge transformations along $\mathcal{D}_j^{(1)}$ and $\mathcal{D}_j^{(2)}$ through the relation:

$$U_2 = U_1^{-T} := U\,. \tag{3.21}$$

The adjoint field $\Phi$ arises from strings stretching between D4-branes of the same stack (i.e. between some $D_j^{(\alpha)}$ and some $D_k^{(\alpha)}$). These give adjoint chiral superfields $\Phi_\alpha$ on the worldvolumes of the two stacks, which are identified by the orientifold projection as:

$$\Phi_2 = \Phi_1^T := \Phi\,. \tag{3.22}$$

The symmetric and antisymmetric fields $S, A$ arise from strings stretching from $\mathcal{D}_j^{(1)}$ to $\mathcal{D}_k^{(2)}$ and backwards. The zero-modes of such strings give rise to massless fields $\Phi_{12}$ and $\Phi_{21}$ in the bi-fundamental representation of the unprojected gauge group U($N$) $\times$ U($N$). Then the orientifold projection imposes:

$$\Phi_{12} = -\Phi_{12}^T := A \qquad \text{and} \qquad \Phi_{21} = \Phi_{21}^T = S\,. \tag{3.23}$$

The identification (3.21) shows that $A$ and $S$ transform in the antisymmetric and conjugate symmetric representations of U($N$). Finally, the eight fundamentals $Q_f$ arise from strings stretching between $\mathcal{D}_j^{(\alpha)}$ and the eight half D6-branes.

As explained in [11], this brane configuration also produces a superpotential of type (2.1). Since we allow the outer NS5-branes to be curved, we have a general polynomial contribution $\mathrm{tr}\, W(\Phi)$ to the superpotential for the adjoint field, where $W$ controls the shape of the outer NS5-branes (this is a minor generalization with respect to [11], which considered tilting but no bending of the NS5-branes ).

To check matching of RR-charges through T-duality, remember that the pair (O6-plane, eight half D6-branes) in the Hanany-Witten realization contains an orientifold plane which is divided in two halves by the central NS5-brane. One half carries D6-brane charge $+4$, while the other half carries D6-brane charge $-4$. This creates a jump in RR charge density along the central NS5-brane which is compensated by the eight half D6-branes. In the geometric engineering construction this corresponds to cancellation of twisted RR-tadpoles (in the twisted sector of the $\mathbb{Z}_2$ orbifold subgroup of the $\mathbb{Z}_4$ orientifold group). Notice that the ansatz (3.18) for the dual coordinates predicts a single orientifold plane in the IIB dual — this cannot be divided into halves as in the Hanany-Witten description, because the NS5 branes are eliminated when performing the T-duality. On the other hand, it is clear that the T-duals of the half D6-branes cannot be ordinary branes, but must be fractional in some sense.[3] This is why the IIB construction involves a further $\mathbb{Z}_2$ orbifold (implemented by the $\mathbb{Z}_2$ subgroup of the orientifold group generated by $\hat{\kappa}^2$). This $\mathbb{Z}_2$ orbifold appears due to taking the T-duality orthogonal to the NS branes but along the O6 plane. This allows us to take the dual D5-branes to be fractional with respect to this $\mathbb{Z}_2$ subgroup, which matches the fact that the central D6 branes in the Hanany-Witten construction carry only half of the usual RR charge of a D6 brane. It is remarkable that the simple ansatz (3.18) (which was already tested in [19] in a different context) maps the $\mathbb{Z}_2$ orientifold of the Hanany-Witten construction into a $\mathbb{Z}_4$ orientifold in IIB, thus automatically implementing this fractionality requirement for the central D5 branes.

Relation (3.13) gives the following action for the $\mathbb{Z}_2$ orbifold generator $\hat{\kappa}^2$:

$$\hat{\kappa}^2 : \quad (z_1, x_1, u_1) \rightarrow (z_1, -x_1, -u_1)\,. \tag{3.24}$$

Using (3.16), we find that the fixed locus of this action coincides with $\hat{C}_1$. Since our O5-plane sits along this locus, we can view it as a fractional orientifold plane, i.e. a $\mathbb{Z}_2$ orientifold with fractional charge under the orbifold action (3.24). Indeed, one can resolve the $\mathbb{Z}_2$ orbifold singularity induced by (3.24) by performing the blow-up of $\hat{X}$ along the curve $\hat{C}_1$ (this amounts to blowing up the point $p_j$ in every fiber $\hat{X}(z)$). Then our fractional O5-plane becomes an O7-plane wrapping the locus $\mathbb{P}(N^*_{\hat{C}_1/\hat{X}})$, which is the $\mathbb{P}^1$ fibration over $\hat{C}_1$ 'glued in' through this resolution. To cancel the charge of this O7 plane, one must add eight D7 branes wrapping the same locus, which correspond to our eight fractional D5-branes after blowing down to recover the original space $\hat{X}$. The strings stretched between these fractional D5-branes and the D5-branes wrapping the exceptional divisors $\mathcal{D}_j^{(\alpha)}$ give rise to the chiral multiplets in the fundamental representation of $\mathrm{U}(N)$. The $\mathrm{SO}(8)$ gauge

[3]The D6 branes span a half of the T-duality circle. This is reminiscent of the fractional D3 branes at conifold singularity, where they become D4 branes on a semicircle after a T-duality.

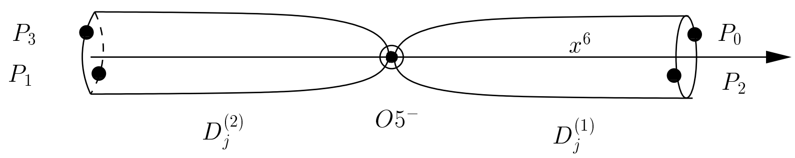


**Figure 4:** The IIB orientifold action $\hat{\kappa}$ along the locus $D_j^{(1)} \cup D_j^{(2)}$. The figure shows an orbit of $\hat{\kappa}$ consisting of points $P_i = \hat{\kappa}^i(P_0)$, with $i = 0, 1, 2, 3$ and $P_0$ a point on $D_j^{(1)}$.

symmetry along the infinite worldvolume of the fractional D5 branes is frozen and recovers the flavor symmetry which rotates the fundamentals of the effective gauge theory.

**Observation.** It is instructive to consider the IIB orientifold action on the exceptional curves $D_j^{(1)}$ and $D_j^{(2)}$, which are realized in the plumbing description as $S^1$ fibrations over the half-axes in $\mathbb{R}^3$ given by $x^4 = x^5 = 0$ and $x^6$ positive or negative. Along these loci we have $u_1 = 0$ (for $D_j^{(1)}$) and $x_1 = 0$ (for $D_j^{(2)}$) and we can take $x^7 = arg(x_1)$ respectively $x^7 = arg(u_1)$. Then (3.14) acts fiberwise by $x^7 \longrightarrow x^7 + \theta$, while the orientifold $\hat{\kappa}$ fixes $x^4 = x^5 = 0$ and acts on $x^6$ and $x^7$ as:[4]

$$\begin{aligned} x^6 &\longrightarrow -x^6 \\ x^7 &\longrightarrow \left[1 - \mathrm{sign}(x^6)\right] \frac{\pi}{2} - x^7 \,. \end{aligned} \tag{3.25}$$

The action on $x^7$ corresponds to reflection along one of two orthogonal diameters of the $S^1$ fiber, depending on whether one acts on a point of $D_j^{(1)}$ or $D_j^{(2)}$ (figure 4). The square $\hat{\kappa}^2$ acts fiberwise by inversion with respect to the origin of the $S^1$ fiber (this is the action by half-period shifts on the periodic coordinate $x^7$):

$$\hat{\kappa}^2 : \quad (x^4, x^5, x^6, x^7) \longrightarrow (x^4, x^5, x^6, \pi + x^7) \,. \tag{3.26}$$

Notice that the orientifold action does not preserve the $S^1$ orbits used to perform the T-duality. This makes it somewhat nontrivial to implement the T-duality explicitly at the level of conformal field theory (another issue which complicates such an approach is the degeneration of the $S^1$ fibers, which is responsible for the appearance of NS5-branes in the IIA description). It would be interesting to study this system in more detail through CFT methods.

### 3.3 Geometric description of supersymmetric vacua

Let us recall the relevant part of the classical moduli space of supersymmetric vacua, which was discussed in [10]. The vacua of interest are solutions of the D- and F-flatness constraints, with the supplementary assumption that the VEV of $\Phi$ is a normal matrix, i.e. $[\Phi, \Phi^\dagger] = 0$. This condition allows us to eliminate possible baryonic branches, which are irrelevant for our purpose.

[4]For convenience, we scale the radius of the $S^1$ fiber to equal one (remember that we don't know the metrics anyway).

The condition that $\Phi$ is normal means that it can be diagonalized through a unitary gauge transformation. Hence one can take:

$$\Phi = \mathrm{diag}(z_1 1_{N_1}, \ldots, z_d 1_{N_d}) , \tag{3.27}$$

where $N_j$ are nonnegative integers such that $\sum_{j=1}^d N_j = N$ (we use the convention that if some $N_j$ vanishes, then the corresponding eigenvalue does not appear in (3.27)). Then it was showed in [10] that the D- and F-flatness constraints imply that the VEVs of $S, A$ and $Q_f$ must vanish in this family of vacua. The unbroken gauge group in such a vacuum is given by $\prod_{j=1}^d \mathrm{U}(N_j)$, with the convention that $\mathrm{U}(0)$ is the trivial group.

In the Hanany-Witten construction, such vacua correspond to a configuration of the type described above, where each $\mathcal{D}_j^{(\alpha)}$ is viewed as a stack of D4-branes with multiplicity $N_j$. Note that the D4-branes must be located at $x^8 + ix^9 = z_j$. This is required in order to preserve $\mathcal{N} = 1$ supersymmetry. In the T-dual IIB construction, we simply have $N_j$ D5-branes wrapped on each of the exceptional curves $D_j^{(1)}$ and $D_j^{(2)}$.

## 4. The geometric transition and effective superpotential

To describe the geometric transition of our system, we follow [3] by considering the most general log-normalizable deformation of $X_0$

$$xy = (u - t_0(z))(u - t_1(z))(u - t_2(z)) , \tag{4.1}$$

where:

$$\begin{aligned} t_0(z) &= -\frac{2W_1'(z) + W_2'(z)}{3} \\ t_1(z) &= \frac{2W_2'(z) + W_1'(z)}{3} \\ t_2(z) &= \frac{W_1'(z) - W_2'(z)}{3} \end{aligned} \tag{4.2}$$

and $W_1, W_2$ are two polynomials (note that $t_0(z) + t_1(z) + t_2(z) = 0$). It is not hard to check (see appendix A) that such a deformation preserves the discrete symmetry (3.6) if and only if it has the form:

$$xy = u(u^2 - W'(z)^2 + 2f_0(z)) , \tag{4.3}$$

where $f_0(z)$ is a polynomial of degree at most $d - 1 = \deg W' - 2$. The deformed space has a 3-section $\Sigma$ (the deformation of $\Sigma_0$) given by $x = y = 0$. This has a rational component $C_1$ with equation $u = 0$ and a hyperelliptic piece $\Sigma^{red}$ given by:

$$\Sigma^{\mathrm{red}} : \quad u^2 - W'(z)^2 + 2f_0(z) = 0 . \tag{4.4}$$

After the geometric transition $\hat{X} \longrightarrow X_0 \longrightarrow X$ of [3], the D5-branes wrapping the exceptional fibers of $\hat{X}$ are replaced by fluxes through the $S^3$ cycles created by smoothing, and the orientifold plane is replaced by the fixed point set $O$ of the action (3.6) on the deformed space $X$. This is given by $x = y = u = 0$, which is the curve $C_1$. The D5-branes wrapped over the orientifold fixed locus also survive the transition. Thus we end up with a compactification with NS-NS and R-R fluxes and a $\mathbb{Z}_4$ orientifold which fixes an O5 plane with worldvolume $\mathbb{R}^{1,3} \times C_1$, together with 8 fractional D5-branes wrapping the orientifold fixed 'plane'. As above, the RR charge of the D5-branes cancels the RR charge of the orientifold. The worldvolume of these D5-branes carries an SO(8) symmetry, which is frozen to a global symmetry of the system because the internal part of the D5-brane worldvolume is non-compact. This gives the geometric realization of the SO(8) flavor symmetry, which is unbroken after confinement.

Since the component $C_1 : \quad u = 0$ of the 3-section is unchanged, the transition can be described as the deformation of the degenerate curve $\Sigma_0^{red}$ defined by $u^2 = W'(z)^2$ to the smooth Riemann surface (4.4). As we shall see below, this 'reduced' geometric process also describes the *planar limit* of the geometric transition associated with the SO($N$) theory with symmetric matter. In fact, the reduced Riemann surface (4.4) coincides with the spectral curve governing the strict planar limit of the matrix model associated to such theories via the Dijkgraaf-Vafa correspondence [25]–[27] and [10]. Geometrically, the component $C_1$ of the Riemann surface is a 'spectator' during the geometric transition, with the relevant planar information encoded by the hyperelliptic curve $\Sigma_{\rm red}$.[5] This matches the field theory and matrix model results of [10]. Indeed, it was shown in [10] that the effective superpotential of our chiral theory agrees with that of the SO($N$) model with symmetric matter. The easiest way to see this is to turn on a small positive Fayet-Iliopoulos parameter in the chiral model, an operation which cannot affect the effective superpotential since the latter is protected by holomorphy. Restricting to the vacua of interest (namely those vacua of the chiral theory for which the matrix $\Phi$ is normal) one finds [10] that turning on such a Fayet-Iliopoulos parameter leads to a pattern of gauge symmetry breaking which recovers the SO($N$) theory with symmetric matter as an effective description of our chiral model (the symmetric field $X$ of the SO($N$) model arises as the symmetric part of $\Phi$, which remains massless after turning on the Fayet-Iliopoulos parameter). Then a direct calculation [10] shows that the scales of the two theories agree, a 'miracle' which is due to the particular field contents under consideration. Hence the effective superpotential of the chiral theory for this particular set of vacua must agree with that of the SO($N$) model with symmetric matter, a conclusion which has been verified by computing the two effective superpotentials upon using the Dijkgraaf-Vafa correspondence [10]. The fact that the component $C_1$ of our Riemann surface is not affected by the geometric transition allows us to describe the transition in terms of the spectral curve $\Sigma^{red}$ of the SO($N$) model with symmetric matter. This is the geometric manifestation of the relation observed in [10] between the two field theories.

Of course, the common effective superpotential of these theories receives contributions from both orientable and unorientable planar diagrams. In the associated matrix models, the former correspond to the strict planar limit, while the latter give the subleading contribution in the $1/\hat{N}$ expansion. Recall from [10] that the gaugino superpotential can be

[5]In the M-theory lift of the IIA Hanany-Witten configuration this shows up in that the middle NS-brane remains flat [22, 23].



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expressed as:

$$W_{\text{eff}} = \sum_{j=1}^{d} \left[ N_j \frac{\partial F_0}{\partial S_j} + 4F_1 + \alpha_j S_j \right], \tag{4.5}$$

where $S_j$ are the gaugino condensates after confinement in the $\mathrm{U}(N_j)$ factors and $\alpha_j$ are related to the effective gauge couplings in these factors. Here $F_0$ and $F_1$ are the leading and subleading contributions to the free energy of the holomorphic matrix model associated to our chiral theory:

$$F = F_0 + \frac{g}{\hat{N}} F_1 + O\left( \left( \frac{g}{\hat{N}} \right)^2 \right), \tag{4.6}$$

where $\hat{N}$ describes the size of the various matrices involved. We refer the reader to [10] for the construction and analysis of this matrix model. As noted in [10], the partition function of this model agrees up to a factor with that of the matrix model associated with the $\mathrm{SO}(N)$ theory with symmetric matter, a fact which can be established by comparing the two eigenvalue representations. More precisely, one has the relation [10]:

$$F = F_{SO} - \frac{g}{\hat{N}} \ln 2\,. \tag{4.7}$$

In the chiral theory, the effective superpotential arises from planar diagrams with or without boundaries associated to the fundamentals $Q_f$. The former correspond to the closed string contribution to the free energy after the geometric transition, while the latter arise from unoriented open strings ending on the eight fractional D5-branes in the presence of the orientifold. As argued in [1], such contributions can be computed directly in the twisted model, so they arise from an orientifolded version of Kodaira-Spencer theory [28], which unfortunately does not seem to have been studied. One could also try to apply the ansatz of [24] in order to extract $W_{\text{eff}}$ directly from our geometric realization. However, remember that the RR charge of the O5-plane is canceled (even locally) by the charge of the D5-branes wrapping the orientifold fixed locus. Hence a naive application of the ansatz of [24] predicts a zero contribution to $W_{\text{eff}}$ from our (O5, D5-brane) system! Of course this cannot be correct, since it contradicts the matrix model and field theory results of [10]. The problem can be traced to the fact that the ansatz of [24] treats orientifold planes simply as sources of flux, so it implies that an orientifold plane can be replaced by a system of D-branes of equal RR charge density as far as its contribution to the effective superpotential is concerned. In our model, the D5-branes wrap the orientifold 5-plane such that the RR charge of the system vanishes locally. However, we know that this system must bring a nonzero contribution to the effective superpotential in order to agree with the results of [19]. Also notice that this is a 'degenerate' case in the sense that our orientifold plane has the same worldvolume as the fractional D5 branes. It seems that the proposal of [24] must be modified in order to account for our situation. One might need to relax the assumption that orientifold planes can be treated the same as D5 branes for the purpose of computing the effective superpotential. Such a modified proposal should recover relation (4.5), which was established in [10] through a direct analysis of generalized Konishi anomalies.



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## 5. Engineering of the $\mathrm{SO}(N)/\mathrm{Sp}(N/2)$ theory with symmetric/antisymmetric matter

As explained in [10] and recalled above, our theories are intimately related to the $\mathrm{SO}(N)$ theory with symmetric matter. It is therefore instructive to consider the geometric engineering of such models within the set-up of [19]. We shall present a geometric realization of such theories through certain $\mathbb{Z}_2$ orientifolds of IIB string theory on non-compact Calabi-Yau $A_2$ fibrations in the presence of D5-branes. More precisely, our IIB background contains a (disconnected) O5-'plane' sitting in the minimal resolution of such a fibration, together with D5-branes wrapping the exceptional $\mathbb{P}^1$'s of the resolution. By changing the sign of the RR charge of the orientifold 'plane', the set-up discussed below allows us to engineer both the $\mathrm{SO}(N)$ with symmetric matter and $\mathrm{Sp}(N/2)$ theories with antisymmetric matter. Thus we shall consider these cases simultaneously. The geometric engineering discussed below could be of independent interest in light of the recent analysis of such models within the framework of the Dijkgraaf-Vafa correspondence [25, 26, 27]. We shall also consider the geometric transition of [1, 3, 4] for our backgrounds. As in [19], we find that the orientifold 5-'plane' survives the transition (through it becomes connected on the other 'side'), and therefore it contributes to the gaugino superpotential obtained after confinement in such field theories.

Let us start with the singular $A_1$ fibration $X_{1,0}$ given by:

$$X_{1,0}: \quad xy = (u - W'(z))(u + W'(z))\,. \tag{5.1}$$

This fibration admits the two-section:

$$\Sigma_{1,0}: \quad x = y = 0\,, \qquad (u - W'(z))(u + W'(z)) = 0\,, \tag{5.2}$$

whose irreducible components are the rational curves $u = \pm W'(z)$.

The resolution $\hat{X}_1$ can be described globally as the complete intersection:

$$\begin{aligned} \beta(u - W'(z)) &= \alpha x \\ \alpha(u + W'(z)) &= \beta y \\ (u - W'(z))(u + W'(z)) &= xy \end{aligned} \tag{5.3}$$

in the ambient space $\mathbb{P}^1[\alpha,\beta] \times \mathbb{C}^4[z,u,x,y]$. The exceptional $\mathbb{P}^1$'s sit above the singular points of $X_{1,0}$, which are determined by $x = y = u = 0$ and $z = z_j$, where $z_j$ are the roots of $W'$. We let $D_j$ denote the exceptional $\mathbb{P}^1$ sitting above $z_j$. The resolved space admits the $\mathrm{U}(1)$ action:

$$([\alpha,\beta], z, u, x, y) \longrightarrow ([e^{-i\theta}\alpha, \beta], z, u, e^{i\theta}x, e^{-i\theta}y)\,, \tag{5.4}$$

which will be of interest below.

We next add an orientifold. Consider the holomorphic $\mathbb{Z}_2$ action:

$$\hat{k}_1: \quad ([\alpha,\beta], z, u, x, y) \longrightarrow ([-\beta,\alpha], z, -u, y, x)\,, \tag{5.5}$$

which is a symmetry of $\hat{X}_1$ stabilizing each exceptional curve $D_j$. It projects to the following



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involution of $X_{1,0}$:

$$\kappa_1 : \quad (z,u,x,y) \longrightarrow (z,-u,y,x)\,, \tag{5.6}$$

whose fixed point set is given by:

$$O_{1,0} : x = y\,, \qquad u = 0\,, \qquad x^2 + W'(z)^2 = 0\,. \tag{5.7}$$

The locus $O_{1,0}$ is a reducible curve with rational components $x = \pm i W'(z)$, which touch each other at the points $(x,z) = (0, z_j)$. The fixed point set $\hat{O}_1$ of (5.5) is the disjoint union of the proper transforms of these curves:

$$\hat{O}_1 : \quad x - y = u = x^2 + W'(z)^2 = 0\,, \qquad \frac{\alpha}{\beta} = \pm i\,. \tag{5.8}$$

Thus the orientifold action on the resolved space determines a (disconnected) orientifold 5-'plane'.

It is not hard to check that this construction engineers the SO$(N)$ or Sp$(N/2)$ theory with symmetric or antisymmetric matter, respectively. The two cases arise by taking the O5 'plane' to have positive/negative RR charge. The field content can be extracted geometrically or by a fractional brane construction. Alternatively, one can use T-duality to map our IIB background to one of the known Hanany-Witten realizations of these field theories [29], and we shall use this approach below. This is essentially an application of [14], supplemented with the ansatz proposed in [19] for the T-dual coordinates.

### 5.1 The T-dual configuration

To extract the T-dual Hanany-Witten configuration, we use a local description valid on a subset $\tilde{X}_1 \subset \hat{X}_1$. In the present case, it is given by two copies $U_0$ and $U_1$ of $\mathbb{C}^3$ with coordinates $(x_i, u_i, z_i)$ $(i = 0, 1)$ which are glued together according to:

$$(x_1, u_1, z_1) = \left(\frac{1}{u_0}, x_0 u_0^2 - 2W'(z_0) u_0, z_0\right). \tag{5.9}$$

The resolution map $\tau$ is given by:

$$\begin{aligned}(z,u,x,y) &= (z_0, x_0 u_0 - W'(z_0), x_0, u_0(x_0 u_0 - 2W'(z_0)))\,,\\ &= (z_1, x_1 u_1 + W'(z_1), x_1(x_1 u_1 + 2W'(z_1)), u_1)\,.\end{aligned} \tag{5.10}$$

The U(1) action (5.4) takes the form:

$$(z_i, u_i, x_i) \longrightarrow (z_i, e^{-i\theta} u_i, e^{i\theta} x_i)\,, \tag{5.11}$$

with fixed point set given by the union of rational curves $x_0 = u_0 = 0$ and $x_1 = u_1 = 0$. It stabilizes the exceptional curves $D_j : \ x_0 = u_1 = z - z_j = 0$ of the resolved fibration.

The Hanany-Witten construction is obtained by T-duality with respect to the circle orbits of (5.11). Following [19], we shall use the following ansatz for the T-dual coordinates:

$$x^4 + ix^5 = x_0 u_0 - W'(z_0) = x_1 u_1 + W'(z_1)\,, \quad x^6 = \frac{1}{2}(|x_1|^2 - |u_0|^2)\,, \quad z = x^8 + ix^9 \tag{5.12}$$

together with the periodic coordinate $x^7$ along the orbits of the U(1) action (5.11).

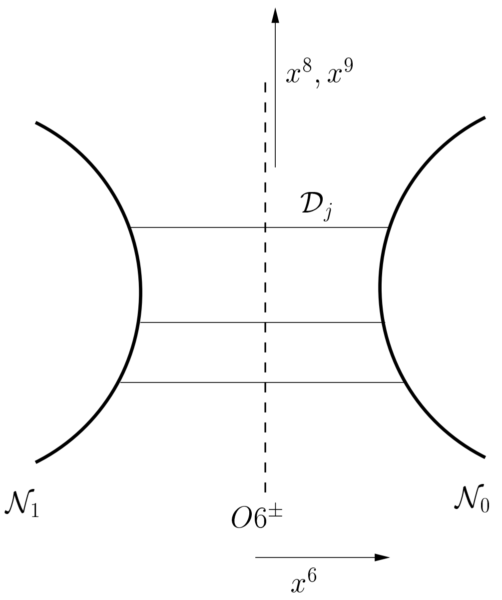


**Figure 5:** Brane configuration for the $\mathrm{SO}(N)/\,\mathrm{Sp}(N/2)$ theories with symmetric/antisymmetric matter. The outer NS5-branes are bent in the directions $x^4$ and $x^5$, which cannot be shown properly in this two-dimensional figure. The $\mathrm{SO}(N)/\,\mathrm{Sp}(N/2)$ gauge groups correspond to positive/negative charge of the orientifold 6-plane.

Expressing the fixed point set of (5.11) in the coordinates (5.12), we find that the T-dual background contains two NS5-branes $\mathcal{N}_0$ and $\mathcal{N}_1$ sitting at:

$$\mathcal{N}_0: \quad x^4 + ix^5 = -W'(z)\,, \qquad x^6 = +\infty \tag{5.13}$$

and:

$$\mathcal{N}_1: \quad x^4 + ix^5 = +W'(z)\,, \qquad x^6 = -\infty\,, \tag{5.14}$$

as well as D4-branes $\mathcal{D}_j$ stretching between the NS5-branes at $z = z_j$.

The orientifold (5.5) acts in local coordinates as:

$$(z_0, x_0, u_0) \longleftrightarrow (z_1, u_1, -x_1)\,. \tag{5.15}$$

The fixed point set is $u_0^2 + 1 = x_0 + W'(z)u_0 = 0$, which is the union of two disjoint rational curves mentioned above. Using (5.12) we find that under T-duality this locus maps to an O6-plane sitting at $x^4 = x^5 = x^6 = 0$ (figure 5). Notice the simple form of the dual O6-plane, despite the fact that the original O5-plane in the resolved space $\hat{X}_1$ has two disconnected components. This is due to nonlinearity of the map (5.12).

The brane configuration of figure 5 is one of the Hanany-Witten systems realizing the SO / Sp theories with one symmetric/antisymmetric chiral multiplet. This establishes the fact that the IIB background considered above engineers these theories.

### 5.2 Description after the geometric transition

After the geometric transition of [1, 3, 4], the Calabi-Yau space (5.1) is deformed to:

$$X_{1,0}: \quad xy = u^2 - W'(z)^2 + 2f_0(z)\,, \tag{5.16}$$

where $f_0(z)$ is a polynomial of degree at most $d-1$. This fibration admits the two-section:

$$\Sigma_1: \quad x = y = 0\,, \qquad u^2 - W'(z)^2 + 2f_0(z) = 0\,, \tag{5.17}$$

which coincides with the reduced component (4.4) obtained after transition in the chiral theory. The D5-branes wrapping the exceptional divisors of the resolution are replaced by fluxes, but the deformed space (5.16) is still invariant under the $\mathbb{Z}_2$ action (5.6) so the orientifold 5-plane survives the transition. Its internal part is deformed to the irreducible curve:

$$O_1: \quad x = y\,, \qquad x^2 + W'(z)^2 - 2f_0(z) = 0\,. \tag{5.18}$$

The Riemann surface (5.17) arises naturally in the confining phase of the SO($N$)/ Sp($N/2$) theory with symmetric/antisymmetric matter [25]–[27] and [10]. As explained in [26, 27], this curve can be extracted by analyzing the generalized Konishi anomalies of such theories, and coincides with the spectral curve which governs the large N limit of the matrix model associated to them via the Dijkgraaf-Vafa correspondence. As in [19], the orientifold 5-'plane' in our geometric engineering survives the transition and brings a nontrivial contribution to the effective superpotential.

## 6. Conclusions

We discussed the geometric engineering and large N transition for the chiral $\mathcal{N} = 1$ field theory containing one adjoint, one antisymmetric, one conjugate symmetric and eight fundamental chiral multiplets. This turned out to be considerably more complex then the much better studied case of non-chiral models. Beyond D5-branes wrapping the exceptional curves of a resolved $A_2$ fibration, our set-up involves the introduction of a $\mathbb{Z}_4$ orientifold together with eight fractional D5-branes required by cancellation of RR tadpoles. This corresponds to cancellation of the chiral anomaly by the eight fundamentals of the field theory model.

Although it is straightforward to perform the large N transition for such backgrounds, it turns out that the $\mathbb{Z}_4$ orientifold and fractional D5 branes survive the transition, and it is far from obvious how to compute their contributions to the effective superpotential by string theory methods. This seems to require a nontrivial modification of the ansatz of [24], or perhaps a more systematic approach through Kodaira-Spencer theory [28]. At the moment we have to rely on the field theoretic arguments of [10], which determine the effective superpotential through a direct analysis of generalized Konishi anomalies and express it in terms of a holomorphic matrix model as expected from the Dijkgraaf-Vafa correspondence.

There are a couple of aspects of our construction which deserve further study. In section 3 we used local RR charge conservation to argue that adding eight fractional D5-branes in our IIB background amounts to canceling RR tadpoles. It would be interesting to check this directly by an explicit study of the tadpole cancellation constraints. Since this is quite nontrivial in our curved background, one could follow the approach of [30] which relies on testing consistency of the worldvolume theory of various D-brane probes.



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One subtle aspect found in [10] is a mismatch between the number of flavors in the field theory and associated holomorphic matrix model. As discussed there, the associated matrix model is consistent only if the number of matrix model flavors equals two. It would be interesting to give a direct derivation of the matrix model of [10] from the B-twisted model associated to our backgrounds.

### Acknowledgments

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## A. Relation to general log-normalizable Calabi-Yau $A_2$ fibrations

Consider a singular $A_2$ fibration of the form (4.1). The symmetry (3.6) is preserved provided that:

$$\prod_{j=0}^{2}(u-t_j(z)) = \prod_{j=0}^{2}(u+t_j(z)) \Longleftrightarrow t_0t_1t_2 \equiv 0\,. \tag{A.1}$$

This leads to the possibilities:

$$\begin{aligned}
t_0 &\equiv 0 \Longleftrightarrow W_1'(z) = -\frac{1}{2}W_2'(z) := W'(z) \Longrightarrow t_2(z) = -t_1(z) = W'(z)\\
t_1 &\equiv 0 \Longleftrightarrow W_2'(z) = -\frac{1}{2}W_1'(z) := W'(z) \Longrightarrow t_2(z) = -t_0(z) = -W'(z)\\
t_2 &\equiv 0 \Longleftrightarrow W_1'(z) = W_2'(z) := W'(z) \qquad \Longrightarrow t_1(z) = -t_0(z) = W'(z)\,.
\end{aligned} \tag{A.2}$$

In each of these cases, we have $t_0t_1 + t_1t_2 + t_2t_0 = -W'(z)^2$ and the hypersurface (4.1) reduces to the form (3.1).

Consider now a general log-normalizable deformation of (4.1):

$$xy = u^3 - p(z)u - q(z)\,, \tag{A.3}$$

where $p(z) = t_0(z)^2 + t_1(z)^2 + t_0(z)t_1(z) - \psi_0(z) - \psi_1(z)$ and $q(z) = -t_0(z)t_1(z)(t_0(z) + t_1(z)) + t_0(z)\psi_1(z) + t_1(z)\psi_0(z) - g(z)$, where $\psi_0, \psi_1$ and $g$ are polynomials of degrees $d-1$. Invariance of (A.3) under (3.6) requires $q \equiv 0$, which together with (A.1) implies:

$$p(z) = W'(z)^2 + f(z)\,, \tag{A.4}$$

where $f(z) := -\psi_0(z) - \psi_1(z)$ is an arbitrary polynomial of degree $d-1$. Relation (A.4) holds in each of the cases (A.2). Thus equation (A.3) reduces to the form (4.3).

## B. Fractional brane construction

For constant $W$, our singular Calabi-Yau threefold becomes the trivial $A_2$ fibration:

$$xy = u^3\,, \qquad z = \text{arbitrary} \tag{B.1}$$

which is simply the product space $\mathbb{C} \times (\mathbb{C}^2/\mathbb{Z}_3)$. We can view this as the orbifold $\mathbb{C}^3/\mathbb{Z}_3$ with action:

$$\rho(\theta) := (z_0, z_1, z_2) \to (z_0, \theta z_1, \theta^{-1} z_2)\,, \tag{B.2}$$

where $\theta = e^{2\pi i/3}$ and $z_0$, $z_1$, $z_2$ are the affine coordinates of $\mathbb{C}^3$. Introducing the invariant coordinates:

$$x = z_1^3\,, \qquad y = z_2^3\,, \qquad u = z_1 z_2\,, \qquad z := z_0 \tag{B.3}$$

the factor $\mathbb{C}^2/\mathbb{Z}_3$ takes the standard form:

$$xy = u^3\,. \tag{B.4}$$

and we recover the description (B.1).

To identify the physics in this limit, let us set $W = \lambda W_0$ and take $\lambda \to 0$. When $\lambda = 0$, the superpotential has the entire $z$-plane as a critical set, and (B.1) can be resolved by double blow up of every fiber. In particular, there is no distinguished point in the base specifying the location of the two D5-branes wrapped over the exceptional $\mathbb{P}^1$'s — we now have two such exceptional curves above every point in the base. Physically, $W$ provides a potential barrier between the states described by given configurations of wrapped D5-branes. In the adiabatic (WKB) limit $\lambda \to 0$, this barrier becomes vanishingly small and tunneling allows the D5-branes to spread out in the $z$ direction. Accordingly, blowing down all fibers after setting $\lambda = 0$ will lead to fractional D3-branes which are delocalized in the $z$ direction, thereby spanning a worldvolume equal to $\mathbb{R}^{1,3} \times \mathbb{C}$. On the other hand, the D5-branes wrapping the rational curve $\hat{C}_1$ survive trivially in this limit, yielding fractional D5-branes with worldvolume $\mathbb{R}^{1,3} \times \mathbb{C}$.

It is instructive to consider this limit in the T-dual IIA description. Remember that $\mathcal{N} = 1$ supersymmetry requires that the D4-branes of the Hanany-Witten configuration are localized at the critical points $z_j$ of $W$ in the direction $z = x^8 + ix^9$ (see figure 3). Since the length of the D4-branes in the direction $x^6$ is related to the size of the exceptional $\mathbb{P}^1$'s of the T-dual model, the singular limit of our $A_2$ fibration corresponds to the outer NS5-branes touching each other at the points $z = z_j$[6] (figure 6). In this limit, all D4-branes become tensionless and can be viewed as codimension two massless excitations inside the five-dimensional boundary of the half D6-branes. Note that the gauge coupling $g_{\rm YM}^2 = g_s$ on the half D6-branes becomes infinite along this boundary, since there is an NS5-brane sitting there. Hence all solitonic excitations of the worldvolume theory of the D6 branes become massless along this boundary, and we can identify the massless D4 branes with solitonic objects confined to the boundary. A nontrivial $W$ provides a potential barrier for the movement of such objects in the $z$ direction, which becomes vanishingly small in the limit $W' \equiv 0$. In this limit, the tensionless D4 branes become delocalized along the boundary of the half D6 branes.

Returning to the IIB picture, consider the field theory along the common part $\mathbb{R}^{1,3}$ of the branes' worldvolume. Since we are mostly interested in the massless field content, it

[6] This is not visible in the coordinates used in subsection 3.2, which are chosen such that the outer NS branes sit at infinity. However, one can redefine these coordinates to place the outer NS5 branes at finite distance.



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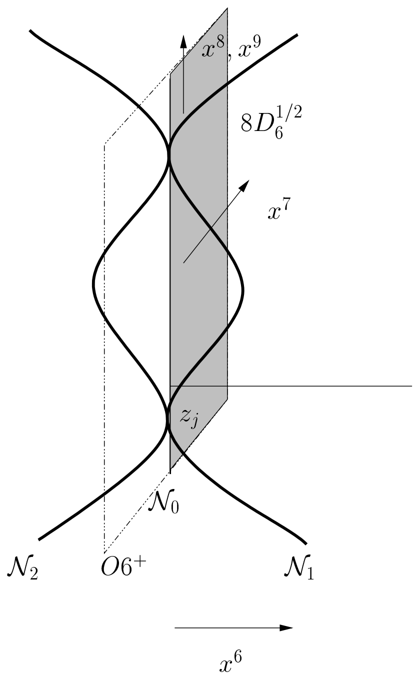


**Figure 6:** Singular limit in the Hanany-Witten picture.

suffices to look at open string modes associated with the coordinates $x^0, \ldots, x^3$, which are insensitive to the fact that the fractional D3 branes are delocalized in the direction $z$. Thus we can use the standard methods of [31] in order to extract the massless field content. For this, let us consider the Chan-Paton representation $R = \oplus_{i=1}^{2} E_i \otimes R_i$, where $E_i$ are some finite-dimensional complex vector spaces and $R_i$ are the irreducible representations of $\mathbb{Z}_3$, i.e. copies of $\mathbb{C}$ carrying the actions:

$$\hat{R}_i(\theta) = \theta^i \,. \tag{B.5}$$

The superfield potential $\mathbf{V}$ and chiral superfields $Z_0, \ldots, Z_2$ of the worldvolume theory transform in the $\mathbb{Z}_3$ representations $\mathrm{End}(R)$ and $\rho \otimes \mathrm{End}(R)$ respectively (here $\rho$ is the geometric representation given in (B.2)). This gives the orbifold projection:

$$\begin{aligned} \mathbf{V}_i{}^j &= \theta^{i-j} \mathbf{V}_i{}^j \\ (Z_1)_i{}^j &= \theta^{i-j+1} (Z_1)_i{}^j \\ (Z_2)_i{}^j &= \theta^{i-j-1} (Z_2)_i{}^j \,, \end{aligned} \tag{B.6}$$

where $\mathbf{V}_i{}^j$ and $(Z_k)_i{}^j$ are the components along $\mathrm{Hom}(E_j, E_i)$. The surviving components are $\mathbf{V}_i{}^i$, $(Z_0)_i{}^i$ and $(Z_1)_1{}^2$, $(Z_2)_2{}^1$.

We next consider the branes wrapping $\hat{C}_1$. Since these are T-dual to the half D6-branes of the Hanany-Witten construction, they should transform nontrivially under the action of the square root $\xi := \theta^{1/2} = e^{i\pi/3}$, which generates a $\mathbb{Z}_6$ group. We implement this by considering the Chan-Paton representation $\Gamma := F \otimes \Gamma_3$, where $F$ is some finite-dimensional

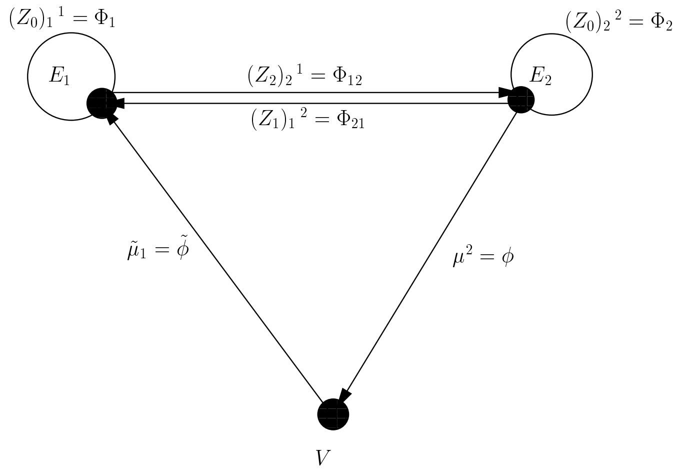


**Figure 7:** The quiver representation before orientifolding. Here $\Phi_{ij}$ etc are the notation used in appendix C.

complex vector space and:

$$\hat{\Gamma}_i(\xi) = \xi^i \tag{B.7}$$

give the the irreps $\Gamma_i$ of $\mathbb{Z}_6$. In terms of the $\mathbb{Z}_6$ generator, the action (B.2) takes the form:

$$\tilde{\rho}(\xi) : (z_0, z_1, z_2) \to (z_0, \xi^2 z_1, \xi^4 z_2)\,. \tag{B.8}$$

This is a non-effective $\mathbb{Z}_6$ action on $\mathbb{C}^3$, with a trivially acting $\mathbb{Z}_2$ subgroup generated by $\xi^3 = -1$. The original D5-branes transform in the $\mathbb{Z}_6$ representations $R_1 = \Gamma_2$ and $R_2 = \Gamma_4$, while $\mathbb{Z}_i$ have the geometric transformations (B.8).

After introducing the new branes, we have chiral superfields $\mu, \tilde{\mu}$ with Chan-Paton representations $\mathrm{Hom}(R, \Gamma)$ and $\mathrm{Hom}(\Gamma, R)$, as well as fields in the representation $\mathrm{End}(\Gamma)$. The latter will be unimportant for our purpose since they live on the central D5-brane, whose worldvolume in the internal directions remains infinite even after turning on the superpotential $W$ and resolving the singular fibers. This means that such fields will be frozen to the values specified by their equations of motion. For the fields $\mu, \tilde{\mu}$ we shall consider the geometric transformations:

$$(\mu, \tilde{\mu}) \to (\xi^{-1}\mu, \xi^{-1}\tilde{\mu}) \tag{B.9}$$

under the action (B.8). This gives the $\mathbb{Z}_6$ projections:

$$\begin{aligned} \mu &= \xi^{-1}\hat{\Gamma}(\xi)\mu\hat{R}(\xi^2)^{-1} \\ \tilde{\mu} &= \xi^{-1}\hat{R}(\xi^2)\tilde{\mu}\hat{\Gamma}(\xi)^{-1}\,, \end{aligned} \tag{B.10}$$

with the solution:

$$\mu = \begin{bmatrix} 0 & \mu^1 \end{bmatrix}, \qquad \tilde{\mu} = \begin{bmatrix} \tilde{\mu}_2 \\ 0 \end{bmatrix}, \tag{B.11}$$

where $\mu^1 \in \mathrm{Hom}(E_1, F)$ and $\tilde{\mu}_2 \in \mathrm{Hom}(F, E_2)$. The surviving fields form the quiver representation shown in figure 7. This recovers the desired field content in the absence of the orientifold projection, in agreement with the work of [20].

We next consider a $\mathbb{Z}_4$ orientifold of this configuration, whose geometric action is given by:

$$(z_0, z_1, z_2) \to (z_0, -z_2, z_1)\,. \tag{B.12}$$

In the invariant coordinates (B.3), this becomes $(z, u, x, y) \to (z, -u, -y, z)$, which agrees with the action used above for geometric engineering. This orientifold makes sense only if $\dim E_1 = \dim E_2 := N$, which assume from now on.

For the Chan-Paton actions, we take:

$$\gamma = \begin{bmatrix} 0 & \mathbf{1}_N \\ \mathbf{1}_N & 0 \end{bmatrix}, \qquad \eta = 1_{N_F}\,, \tag{B.13}$$

where $N_F := \dim F$. The orientifold projections are:

$$\begin{aligned} \mathbf{V} &= -\gamma \mathbf{V}^T \gamma^{-1} \\ Z_0 &= \gamma Z_0^T \gamma^{-1} \\ Z_1 &= -\gamma Z_1^T \gamma^{-1} \\ Z_2 &= \gamma Z_2^T \gamma^{-1} \\ \tilde{\mu} &= \gamma \mu^T \eta^{-1}\,. \end{aligned} \tag{B.14}$$

This orientifold action exchanges nodes $E_1$ and $E_2$ of the quiver, while fixing the node labeled $F$. Writing it in components, we find (figure 8):

$$\begin{aligned} {\mathbf{V}_2}^2 &= -[{\mathbf{V}_1}^1]^T := \mathbf{V}\,, & {(Z_0)_2}^2 &= [{(Z_0)_1}^1]^T := \Phi \\ {(Z_1)_1}^2 &= -[{(Z_1)_1}^2]^T := A\,, & {(Z_2)_2}^1 &= [{(Z_2)_2}^1]^T := S \\ \tilde{\mu}_2 &= -(\mu^1)^T := Q \end{aligned} \tag{B.15}$$

One can also consider the vector fields $\nu \in \mathrm{End}(\Gamma, \Gamma)$ on the central brane, with the trivial orbifold projection and the orientifold projection $\nu = -\eta \nu^T \eta^{-1}$. This shows that the gauge group on the worldvolume of the central brane is projected to $\mathrm{SO}(N_F)$. Since this brane has infinite volume even after resolving the singularities and turning on $W$, its gauge symmetry is frozen to a global $\mathrm{SO}(N_F)$ invariance of the effective action of our system.

The first equation in (B.15) implies that the gauge transformations $U_1$ and $U_2$ are identified as $U_2 = U_1^{-T} := U$. This produces the desired $\mathrm{U}(N)$ gauge symmetry. We are left with the antisymmetric and symmetric fields $A$, $S$, the adjoint field $\Phi$ and the $N_F$ fundamental fields given by the components of $Q \in \mathrm{Hom}(W, E_1)$ along a basis of $W$. Because the gauge group elements associated to $E_0$ and $E_1$ are identified as $U_2 = U_1^{-T} := U$, the fields $A$ and $S$ transform in the antisymmetric and conjugate symmetric representations of $\mathrm{U}(N)$. It is easy to check that the only cubic terms in the superpotential which involve $Q, S$ or $A$ and are consistent with this field content and gauge invariance are of the form $\mathrm{tr}(Q^T S Q)$ and $\mathrm{tr}(S \Phi A)$. Of course, RR tadpole cancellation requires $N_F = 8$, as argued independently in section 3.



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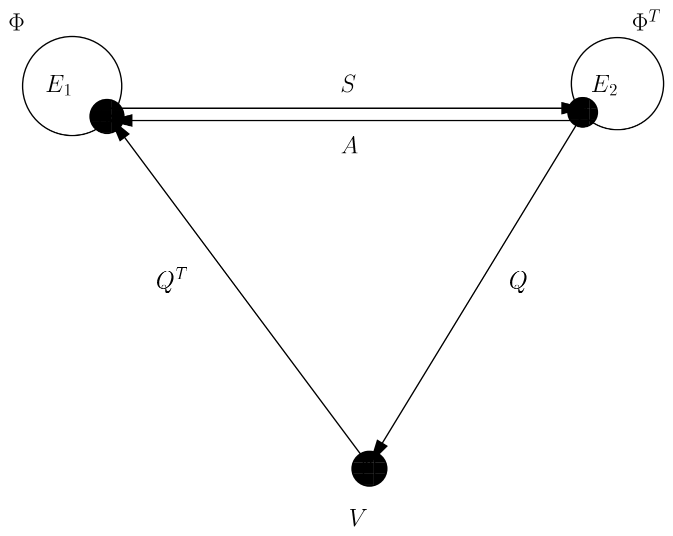


**Figure 8:** Projected quiver representation.

We also note that an antisymmetric choice

$$\gamma = \begin{bmatrix} 0 & \mathbf{1}_N \\ -\mathbf{1}_N & 0 \end{bmatrix} \tag{B.16}$$

is physically equivalent. The differences are that with this choice the gauge transformations are identified according to $U_1 = U_2^T$ and that $Z_2$ projects onto the symmetric field $S$ whereas $Z_1$ projects onto the antisymmetric field $A$. In the T-dual Hanany-Witten construction this corresponds to the choice of having positive O6-plane charge along the positive or negative $x^7$ direction.

## C. Engineering without a tree level potential

Let us now sketch the geometric interpretation of the fractional brane construction. Consider the toric resolution $(\mathbb{C}^4 - \mathcal{Z})/(\mathbb{C}^*)^2$ of the $A_2$ singularity $\mathbb{C}^2/\mathbb{Z}_3$, with charge matrix:

$$Q = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \tag{C.1}$$

and toric generators given by the columns of:

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}. \tag{C.2}$$

The generators correspond to homogeneous coordinates which we denote by $w_1, \ldots, w_4$. The exceptional set is $\mathcal{Z} = \{w_1 = w_3 = 0\} \cup \{w_2 = w_4 = 0\} \cup \{w_1 = w_4 = 0\}$. We let $D_j = (w_j)$ be the toric divisors. Then $D_2 = D^{(1)}$ and $D_3 = D^{(2)}$ are the exceptional $\mathbb{P}^1$'s, while $D_1$ and $D_4$ are non-compact.

In the symplectic quotient description, we have the reduction of $\mathbb{C}^4$ with respect to the $\mathrm{U}(1)^2$ action defined by (C.1) with moment map equations:

$$\begin{aligned} |w_1|^2 - 2|w_2|^2 + |w_3|^2 &= \zeta_1 \\ |w_2|^2 - 2|w_3|^2 + |w_4|^2 &= \zeta_2 \,, \end{aligned} \tag{C.3}$$

where $\zeta_j$ are some positive levels. Setting $\zeta_1 = \zeta_2 = 0$ recovers the $A_2$ singularity, which is described by the invariants:

$$\begin{aligned} x &= w_1^3 w_2^2 w_3 \\ y &= w_2 w_3^2 w_4^3 \\ u &= w_1 w_2 w_3 w_4 \,, \end{aligned} \tag{C.4}$$

subject to the relation $xy = u^3$.

The orientifold action on the minimal resolution takes the form:[7]

$$(w_1, w_2, w_3, w_4) \longrightarrow (w_1, w_3, -w_2, w_4) \,. \tag{C.5}$$

The only fixed fixed point is $p := [1, 0, 0, 1]$, which is the intersection of $D_2$ and $D_3$. We have the intersections:

$$D_1 \cap D_2 = \{q\} = \{[0, 0, 1, 1]\} \,, \qquad D_4 \cap D_3 = \{r\} = \{[1, 1, 0, 0]\} \,. \tag{C.6}$$

As expected, the action (C.5) permutes the compact divisors.

Consider two D5-branes along $\mathbb{R}^{1,3} \times D_2$ and $\mathbb{R}^{1,3} \times D_3$ with trivial Chan-Paton bundles $\mathcal{E}_1$ and $\mathcal{E}_2$ such that $\hat{\kappa}^*(\mathcal{E}_1) \approx \mathcal{E}_2$ and $\hat{\kappa}^*(\mathcal{E}_2) \approx \mathcal{E}_1$. Then the orbifold action $\hat{\kappa}^2$ maps $\mathcal{E}_j$ into itself in the sense that $(\hat{\kappa}^*)^2(\mathcal{E}_j) \approx \mathcal{E}_j$. We also consider a D5-brane with worldvolume $\mathbb{R}^{1,5} \times \{p\}$ , which carries a Chan-Paton bundle $\mathcal{F}$. The massless states of the target space theory arise along the locus $\mathbb{R}^{1,3} \times \{p\}$, which is the intersection of all worldvolumes. Therefore, it suffices to concentrate on the fibers $E_j$, $F$ of $\mathcal{E}_j$ and $\mathcal{F}$ at the point $p$. The massless fields are associated with morphisms $\Phi_i \in \mathrm{End}(E_i)$ and $\Phi_{12} \in \mathrm{Hom}(E_1, E_2)$, $\Phi_{21} \in \mathrm{Hom}(E_2, E_1)$ as well as $\phi_1 \in \mathrm{Hom}(E_1, F)$, $\phi_2 \in \mathrm{Hom}(E_2, F)$ and $\tilde{\phi}_1 \in \mathrm{Hom}(F, E_1)$, $\tilde{\phi}_2 \in \mathrm{Hom}(F, E_2)$.

For simplicity, we shall endow the vector spaces $E_j$ and $V$ with hermitean metrics (this is not strictly necessary, but it allows us to formulate the orientifold projection in a more traditional manner). Then the orientifold action is implemented by invertible maps:

$$\begin{aligned} &\gamma_0 : F \to F \\ &\gamma_1 : E_1 \to E_2 \\ &\gamma_2 : E_2 \to E_1 \,. \end{aligned} \tag{C.7}$$

Let us first consider the wrapped D5 branes. In this sector we have $\gamma_1 \gamma_2^{-T} = \mathbf{1}_{E_1}$.[8] Using $\gamma_1$ to identify $E_1$ and $E_2$ to a vector space we call $E$, we can set $\gamma_1 = \gamma_2 = \mathbf{1}_E$. Then

[7] In the symplectic quotient description, this is a symmetry if we set $\zeta_1 = \zeta_2 = \zeta$.
[8] Using $\gamma_1 \gamma_2^{-T} = -\mathbf{1}_{E_1}$ leads to equivalent results.

the orientifold projection takes the form (B.15):

$$\begin{aligned}\Phi_2 &= \gamma_1 \Phi_1^T \gamma_1^{-1} = \Phi_1^T := \Phi \\ \Phi_{21} &= \gamma_2 \Phi_{21}^T \gamma_1^{-1} = \Phi_{21}^T := S \\ \Phi_{12} &= -\gamma_1 \Phi_{12}^T \gamma_2^{-1} = -\Phi_{12}^T := A\,.\end{aligned} \tag{C.8}$$

The orientifold projection on the vector fields $\mathbf{V}_1$ and $\mathbf{V}_2$ on the two wrapped D5 branes is:

$$\mathbf{V}_2 = -\gamma_1 \mathbf{V}_1^T \gamma_1^{-1} = -\mathbf{V}_1^T := \mathbf{V}\,. \tag{C.9}$$

This shows that the U($N$) gauge transformations on the wrapped D5 branes are identified as:

$$U_2 = U_1^{-T} := U\,, \tag{C.10}$$

thereby giving a U($N$) gauge group under which $A$ and $S$ transform in the antisymmetric and conjugate symmetric representations.

We next consider the orientifold projection on the D5-branes carrying the trivial Chan-Paton bundle $\mathcal{F} = F \times \hat{C}_1$. Since $\hat{C}_1$ is invariant under the $\mathbb{Z}_2$ orbifold, the space $F$ carries a representation of $\mathbb{Z}_2$ and thus it can be decomposed as $F = F_+ \oplus F_-$, where the $\mathbb{Z}_2$ generator acts as $\pm\mathbf{1}$ on the subspaces $F_\pm$. Because the orientifold generator squares to the $\mathbb{Z}_2$ orbifold generator, the invertible map $\gamma_0$ must fulfill:

$$\gamma_0 \gamma_0^{-T} = \begin{pmatrix} \mathbf{1}_{F_+} & 0 \\ 0 & -\mathbf{1}_{F_-} \end{pmatrix}. \tag{C.11}$$

One can choose:

$$\gamma_0 = \begin{pmatrix} \gamma_{0,s} & 0 \\ 0 & \gamma_{0,a} \end{pmatrix} \tag{C.12}$$

with $\gamma_{0,s}\gamma_{0,s}^{-T} = \mathbf{1}_{F_+}$ and $\gamma_{0,a}\gamma_{0,a}^{-T} = -\mathbf{1}_{F_-}$. Considering the vector field $\mathbf{V}$ living on the fractional D5-branes, the $\mathbb{Z}_2$ orbifold projection implies that only the diagonal sectors $\mathbf{V}^+ = \mathbf{V}^{++}$ and $\mathbf{V}^- = \mathbf{V}^{--}$ survive. Then the orientifold projection gives:

$$\begin{aligned}\mathbf{V}^+ &= -\gamma_{0,s}(\mathbf{V}^+)^T \gamma_{0,s}^{-1} \\ \mathbf{V}^- &= -\gamma_{0,a}(\mathbf{V}^-)^T \gamma_{0,a}^{-1}\,.\end{aligned} \tag{C.13}$$

The result is an SO gauge group in the + sector and an Sp gauge symmetry in the − sector. Thus we obtain two types of fractional D5 branes, which carry SO respectively Sp gauge symmetry on their worldvolume. To cancel the RR tadpoles, we need eight more fractional D5-branes in the sector + than in the sector −. In the Hanany-Witten picture, the choice (C.12) corresponds to a more general configuration which contains half D6 branes stretching in the positive $x_7$ direction as well as half D6 branes stretching in the negative $x_7$ direction. The first are T-dual to fractional D5 branes in the sector +, while the second correspond to fractional D5-branes in the sector −. Notice that one can form bound states of pairs of fractional branes belonging to the two sectors, giving whole D5-branes which can move away from the $\mathbb{Z}_2$ orbifold fixed locus $\hat{C}_1$. In the Hanany-Witten construction, this corresponds to the recombination of a half D6 brane stretching in the positive $x_7$ direction with a half D6 brane stretching in the negative $x_7$ direction to a whole D6-brane which can move away from the central NS5-brane. In the case of interest for us, one has eight fractional D5-branes in the sector + and no fractional D5 branes in the sector $-$. Accordingly, we take $F_+ \approx \mathbb{C}^{N_F}$, $F_- = 0$ as well as $\gamma_{0,s} = \mathbf{1}_{N_F}$ with $N_F = 8$.

Finally, we have the massless fields $\phi_1$, $\tilde{\phi}_1$, $\phi_2$ and $\tilde{\phi}_2$ corresponding to $\mathrm{Hom}(E_1, F)$, $\mathrm{Hom}(F, E_1)$, $\mathrm{Hom}(E_2, F)$ and $\mathrm{Hom}(F, E_2)$. They transform as follows under the action of the orientifold generator:

$$\begin{aligned}
\phi_1 &\to -\gamma_{0,s}\tilde{\phi}_2^T \gamma_2^{-1} = -\tilde{\phi}_2^T \\
\phi_2 &\to -\gamma_{0,s}\tilde{\phi}_1^T \gamma_1^{-1} = -\tilde{\phi}_1^T \\
\tilde{\phi}_1 &\to \gamma_2 \phi_2^T \gamma_{0,s}^{-1} = \phi_2^T \\
\tilde{\phi}_2 &\to -\gamma_1 \phi_1^T \gamma_{0,s} = -\phi_1^T \,.
\end{aligned} \tag{C.14}$$

This gives the following action and projections for the $\mathbb{Z}_2$ orbifold generator (=the square of the orientifold generator):

$$\begin{aligned}
\phi_1 &\to -\tilde{\phi}_2^T \to \phi_1 \\
\phi_2 &\to -\tilde{\phi}_1^T \to -\phi_2 \Longrightarrow \phi_2 = 0 \\
\tilde{\phi}_1 &\to \phi_2^T \to -\tilde{\phi}_1 \Longrightarrow \tilde{\phi}_1 = 0 \\
\tilde{\phi}_2 &\to -\phi_1^T \to \tilde{\phi}_2 \,.
\end{aligned} \tag{C.15}$$

Hence only $\phi := \tilde{\phi}_2 = -\phi_1^T$ survives, giving eight fundamentals of the $\mathrm{SU}(N)$ gauge group on the wrapped D5-branes. Thus we recover the massless field content extracted in appendix B.[9]

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